Gerbert's letter this rendering is given once. His criticism of Günther's figure is valid only if other interpretations of the Latin involved are invalid. Günther's argument, I take it, is simply this, that $\frac{1}{2}a(a+1)$ squares exceed the $\frac{1}{2}a(a+1)$ rectangles drawn, which in turn exceed the area of the given triangle. Finally, I predict that, if Gerbert's own figure is ever brought to light, it will be found to be not Bubnov's nor Miller's, but Cantor's or Günther's.

RECENT PUBLICATIONS.

REVIEWS.

New Mathematical Pastimes. By P. A. MacMahon. Cambridge, University Press, 1921. 8vo. 7 + 116 pages. Price 12 Shillings.

This is an interesting little volume filled with strange and bizarre figures,[1] and punctuated with quaint quotations in verse.[2] The notions are extremely simple (which may surprise readers of some of the author's more serious works) and the language is clear almost to verbiage, as for example (page 50): "The question now before us is the transformation of the set of pieces so that they will no longer be of the same shape and differently coloured or numbered. It proves to be possible to effect this so that the pieces are of different shapes, and are not differently coloured, numbered or otherwise distinguished. In fact, instead of having a set of pieces of the same shape and differently coloured we construct a set of different shapes but of the same colour. The boundary of the assembled pieces now varies in shape with each type and variety but is not otherwise distinguished." The matter is as a whole fresh and pleasingly consecutive. It is far removed in content and treatment from the advanced texts usually issued by the Cambridge University Press.

The author describes his purpose in the following words: "The author of this book has, of recent years, devoted much time and thought to the development of the subject of 'Permutations and Combinations' with which all students are familiar. He has been led, during that time, to construct, for use in the home circle, various sets of pieces, of elementary geometrical shapes based upon these ideas, and he now for the first time brings them together with the object of introducing, in a wider sphere, what he believes to be a pleasant by-path of mathematics which has almost entirely escaped the attention of the well-known writers upon Mathematical Recreations and Amusements. The book differs *in toto* from their works because everything that it contains, with scarcely an exception, is the invention of the author. It is not a bringing together of materials derived from wholly different ideas. From beginning to end it proceeds along one defined

[1] In its few small pages are one hundred thirty-five separate cuts, averaging some ten distinct designs to a cut.

[2] There are fifty-seven poetic quotations from a wide range of classical sources, scattered through the book. Most of these in their original setting could have had but the most tenuous connection with the thought of this text. They serve merely to entertain the reader by their casual verbal allusions.

path from which it never diverges. One continuous thread of thought runs through it from cover to cover."

The book is divided into three parts. The first two are intimately related and yield a systematic method of constructing certain types of what in America we call "jig-saw puzzles." Those here treated have all of their pieces distinct in shape and possess the peculiar advantage of having a minimum number of distinct types of edges. The age-old and ever fresh popularity among children of picture cubes and jig-saw puzzles is ample excuse for the interesting and novel study here presented. A map dissection, in which no two pieces will fit together unless they match in the completed mosaic, will amuse youthful minds but gives training and practice in little more than the simplest sort of visual perception. A puzzle offering the maximum number of possible pairings other than the correct one has an added interest proportional to the ingenuity required. Probably most readers of the MONTHLY have seen the celebrated "Chinese" checkerboard puzzle, for which the checkerboard is sectioned into many differently shaped pieces each of which contains several of the original component squares. This deservedly popular incitement to hours of fruitless patience is not mentioned explicitly by the author. The checkerboard puzzle is exasperatingly baffling, but the reviewer feels convinced that for a given number of pieces the systematic treatment by the author bids fair to yield the most ingenious designs. Perhaps some enterprising toy-maker will forthwith attempt to copyright a large number of these and the present generation may yet greet a whole class of fresh puzzles, new in detail but in general character nearly as old as the human race. The author refrains from touching upon related but diverging problems, such as the dissection of a given figure into an assigned number of pieces which may be refitted to form another given figure.

Part III while following readily upon the previous portions of the book has a wider mathematical appeal, and does not in fact require the elaboration of the first two parts for its understanding. It is concerned with the principles of repeating designs. The treatment is synthetic in that it proceeds from the elementary bases of triangles, parallelograms, squares and hexagons to more elaborate figures. In this third part, one meets many interesting and novel designs but the reviewer is personally disappointed at the lack of suggestive references. The subject is obviously identical in content with the analysis of repeated designs in the Euclidean plane. Far from being new, this is one of the classic lines of investigation in connection with elliptic functions. A possible fundamental region for an elliptic function is no other than a figure whose duplicates under translation may be made to fill the plane without overlapping. The celebrated investigations which in a vastly wider domain have applied group theory to transcendental functions (typified for example by Klein's work on the elliptic modular function) have been left unacknowledged and worse still apparently unutilized by the author. A more definite reduction to the standard types of fundamental triangles might have systematized the discussion. The only excuse although it is perhaps an adequate one for this proofless synthetic treatment is the

fact that the work merely claims to be a pastime. The combinatorial character of the first two parts is suggestive of the analogous problem of the magic square. There one does not expect a completely analytic treatment except under favorable restrictions. The third part deals rather with systematic material affected by the presence of arbitrary arcs. It becomes a problem of analysis situs combined perhaps with certain non-scientific esthetic demands of minor interest. Possibly in the more extensive discussion that the author promises the objections here raised may be no longer applicable.

A useful but limited chronological bibliography of works on mathematical recreations is given in the appendix.

The casual reader may be disappointed in finding no explicit statement of puzzles to tax his ingenuity and arouse his interest. The "pastimes" are suggested rather than directly announced. But any one with a little imagination and with a healthy taste for numerical and tactical puzzles will find a fruitful source of amusement in this book. A far more instructive book might perhaps have been developed upon the same basis of original material but no one should demand instruction in any book claiming to offer nothing more than new mathematical pastimes.

ALBERT A. BENNETT.

NOTES.

The excellent *Revue de l'Enseignement des Sciences*, published by Felix Alcan, Paris, 1909–1920, has ceased publication. Its place is taken by *Bulletin Scientifique des Professeurs de l'Enseignement du 2e degré* (*B.S. 2*) which has been published twice a month since October, 1921, at Chaussenueil, France; P. Martin, director; 12 francs a year.

Mr. ROGER S. HOAR, a member of the Association, has printed privately a discussion of the mechanics of a new design of gasoline power shovel. It is in a simple form, convenient for students of elementary mechanics. Any number of copies gratis are to be had upon application to him, care of Bucyrus Company, South Milwaukee, Wis. This suggestion may appeal to some teacher looking for "practical" applications.

Mr. HARRY B. MARSH, head of the department of mathematics in the Technical High School, and Springfield Junior College, Springfield, Mass., has published a pamphlet: *Elementary Algebra Outline based upon College Entrance Requirements and Examination Papers* (New York, Newson and Co., 1922, 48 pages).

A twenty-nine page printed report to the College Entrance Examination Board upon Elementary Algebra, Advanced Algebra, and Plane Trigonometry by the Commission on College Entrance Requirements in Mathematics, 431 West 117th Street, New York City, has recently been distributed. The Commission's Report on Plane Geometry and Solid Geometry is being printed separately. The report is elaborate and specific and should come to the attention of all interested in the ground covered in sub-freshman work in mathematics. Professor W. F. OSGOOD, of Harvard University, is chairman of the Commission.

We have already indicated (*1921*, 267) the contents of the first edition of